A Simple Method for Estimating a Sea-Level Rise Allowance, Accounting for Uncertainty

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Projections of sea-level rise are inherently uncertain, leading to considerable debate over suitable vertical allowances by which future infrastructure would need to be raised. Words such as 'plausible' and 'high-end' abound, with little objective or statistically valid support.

A simple method of determining a future sea-level rise allowance is derived, based on the projected rise in mean sea level and its uncertainty, and on the variability of present tides and storm surges (storm tides). The method preserves the expected frequency of flooding events under a given projection of sea-level rise. It is assumed that the statistics of storm tides relative to mean sea level are unchanged.

The method is demonstrated using the GESLA (Global Extreme Sea-Level Analysis) data set of mainly hourly sea levels. Three possible projections of sea-level rise are assumed for the 21st century: two based on the Third and Fourth Assessment Reports of the Intergovernmental Panel on Climate Change (IPCC) and a larger one based on research since the Fourth Assessment Report.

Using sea-level rise projections from the IPCC, the method indicates an Australian allowance for sea level rise during the 21st century of 0.68 ± 0.04 (sd) m (the spread representing the spatial variation of the allowance). Projections released since the last IPCC report suggest a significantly higher allowance of 1.44 ± 0.07 (sd) m. However, it should be stressed that there is considerable scientific debate about which of these sets of projections is the more realistic.

The form of the distribution function selected for the uncertainty in the sea-level rise projection is crucial; a normal distribution, which is unbounded, most likely overestimates the allowance if the projections are extrapolated into the 22nd century.

1. Introduction

A major effect of climate change is a present and continuing increase in sea level, caused mainly by thermal expansion of seawater and the addition of water to the oceans from melted land ice (e.g. Meehl et al., 2007), as reported in the Fourth Assessment Report (AR4) of the Intergovernmental Panel on Climate (IPCC)). The present rate of global-average sea-level rise is about 3.2 mm yr^{-1} (Church and White, 2011). At the time of AR4 in 2007, sea level was projected to rise at a maximum rate of about 10 mm yr^{-1} and to a maximum level of about 0.8 m (relative to 1990) by the last decade of the 21st century, in the absence of significant mitigation of greenhouse-gas emissions (Meehl et al., 2007: Table 10.7, including 'scale-up ice sheet discharge'). However, since the AR4, there has been considerable debate about whether these projections are underestimates (e.g. Nicholls et al., 2011: Fig. 1), as discussed in Section 3.

Sea-level rise, like the change of many other climate variables, will be expressed mainly as an increase in the frequency or likelihood (probability) of extreme events, rather than simply as a steady increase in an otherwise constant state. One of the most obvious adaptations to sea-level rise is to raise infrastructure (or its protection) by a sufficient amount so that flooding events occur no more often than they did prior to the sea-level rise. The selection of such an *allowance* has often, unfortunately, been quite subjective and qualitative, involving concepts such as 'plausible' or 'high-end' projections.

This paper describes a simple technique for estimating an allowance for sea-level rise using elementary extreme-value theory. This allowance ensures that the expected, or average, number of extreme events in a given period is preserved. In other words, any infrastructure raised by this allowance would experience the same frequency of extreme events under sea-level rise as it would without the allowance and without sea-level rise.

It is assumed here that there is no change in the variability of the extremes (specifically, the *scale parameter* of the Gumbel distribution; see Section 2). In other words, the statistics of storm tides relative to mean sea level are assumed to be unchanged. It is also assumed that there is no change in wave climate (and therefore in wave setup and runup).

The allowance derived from this method depends strongly on the distribution function of the uncertainty in the rise in mean sea level at some future time. However, once this distribution has been chosen, the remaining derivation of the allowance is entirely *objective*.

Unless otherwise stated, uncertainties are here given as ± one standard deviation (indicated by 'sd') or as ± the half-range (indicated by 'lim'). In the latter case, the half-range represents true limits, with zero probability outside the indicated range.

The method, which is fully described in Hunter (2011), is summarized in the following.

2. THEORY

Two statistics are conventionally used to describe the likelihood of extreme events such as flooding from the ocean. These are the *average recurrence interval* or *ARI* (R), and the *exceedance probability* (E) for a given period (T). The ARI is the average period between extreme events (observed over a long period with many events) and is often called the *return period*, while the exceedance probability is the probability of at least one exceedance event happening during the period T. Exceedance distributions are often expressed in terms of the *cumulative distribution function* (F), which is just the probability that there will be *no* exceedances during the prescribed period (T); i.e. $F = 1 - E$. These statistics are related by (e.g. Pugh, 1996):

$$F = 1 - E = \exp\left(-\frac{T}{R}\right) = \exp(-N) \tag{1}$$

where N is the expected, or average, number of exceedances during the period T.

The probability of exceedances above a given level and over a given period is often well described by a *generalised extreme-value distribution (GEV)*. The simplest of these, the *Gumbel* distribution, fits most sea-level extremes quite well (e.g. van den Brink and Konnen, 2011). The Gumbel distribution may be expressed as (e.g. Coles, 2001: 47)

$$F = \exp\left(-\exp\left(\frac{\mu - z_P}{\lambda}\right)\right) \tag{2}$$

where z_P is the physical height (e.g. the height of a critical part of the infrastructure), μ is the *location parameter* and λ is the *scale parameter* (an e-folding distance in the vertical). F is the probability that there will be no exceedances > z_P during the period T.

From Eqs. 1 and 2

$$N = \exp\left(\frac{\mu - z_P}{\lambda}\right) \tag{3}$$

μ is therefore the value of z_P for which N = 1 during the period T.

As noted in Section 1, it is assumed that the scale parameter (λ) does not change with a rise in sea level.

Mean sea level is now raised by an amount $\Delta z + z'$, where Δz is the central value of the estimated rise and z' is a random variable with zero mean and a distribution function ($P(z')$), to be chosen below. This effectively increases the location parameter (μ) by $\Delta z + z'$. At the same time, the infrastructure is raised by an allowance a so that it is now located at a height $z_P + a$. It may be shown (Hunter, 2011) that, under these conditions of (uncertain) sea-level rise *and* raising of the infrastructure, the *overall* (or *effective*) expected number (N_{ov}) of exceedances ($> z_P + a$) during the period T, becomes

$$\begin{aligned} N_{ov} &= \int_{-\infty}^{\infty} P(z') \exp\left(\frac{\mu - z_P + \Delta z + z' - a}{\lambda}\right) dz' \\ &= N \exp\left(\left(\Delta z + \lambda \ln\left(\int_{-\infty}^{\infty} P(z') \exp\left(\frac{z'}{\lambda}\right) dz'\right) - a\right) \Big/ \lambda\right) \end{aligned} \tag{4}$$

In order to preserve the expected number of exceedances (or flooding events), we require that $N_{ov} = N$. Therefore, the allowance (a) is equal to the term $\Delta z + \lambda \ln(\ldots)$ in the last part of Eq. 4. This allowance is composed of two parts: the mean sea-level rise ($\underline{\Delta z}$) and the term $\lambda \ln(\ldots)$, which arises from the *uncertainty* in future sea-level rise. Hunter (2011) evaluated the allowance for three types of uncertainty distribution for future sea-level rise: a normal distribution, a boxcar (uniform) distribution and a raised cosine distribution. The resulting allowances may be all expressed as simple analytical expressions, involving the Gumbel scale parameter (λ), the central value of the estimated rise (Δz) and its standard deviation (σ). The boxcar and raised cosine distributions, which have upper and lower limits, are considered here because there are quite strong physical constraints on sea-level rise. For example, it is highly unlikely that sea level will fall under global warming and Pfeffer et al. (2008) deduced an upper limit of sea-level rise for the 21st century of 2.0 m.

3. PROJECTIONS OF SEA-LEVEL RISE

The sea-level rise allowance described in Section 2 requires an estimate of the mean sea-level rise, (Δz) and the standard deviation of its uncertainty (σ). These estimates may be provided by combining results from the IPCC Assessment Reports (specifically, the Third Assessment Report (TAR; Church at al., 2001) and the AR4 (Meehl et al., 2007)), and from research conducted since the AR4, as summarised for example by Nicholls et al. (2011). These two sources of information (i.e. TAR/AR4 and post-AR4) lead to two rather distinct ranges (Hunter, 2011) and are treated separately in the following discussion. At present, it is unclear which of the two is the more appropriate. The present work uses only projections of *global-average* sea-level rise; regional variation therefore represents additional uncertainty.

The projections described here apply only to the component of sea-level rise that is related to anthropogenic climate change. They do not include any effects of vertical land movement, such as those associated with glacial isostatic adjustment, tectonic activity or local land sinkage. Any such movement, and its uncertainty, should be incorporated into the projections, to yield the sea-level rise relative to the land.

3.1. The TAR and AR4 Projections

The TAR projections were presented as time series from 1990 to 2100, while the AR4 projections were only presented as the sea-level rise for 2090-2099 relative to 1980-1999. In order to obtain time series of model projections through the 21st century that are compatible with the AR4, Hunter (2010) fitted the time series of TAR projections through the AR4 projections for 2090-2099. The resultant tables (Hunter, 2010: Tables 1 and 2) are similar to Table II.5.1 of the TAR and are here referred to as the *AR4-adjusted TAR projections*.

Two sets of sea-level rise allowances are provided for the *AR4-adjusted TAR projections*, both of which are based on the A1FI emission scenario (which the world is broadly following at present; Le Quéré et al., 2009). Both sets use a mean sea-level rise (Δz) based on the average of the 5 and 95% projections, but use different distribution functions ($P(z')$) fitted through the 5 and 95% values:

- the *IPCC A1FI Projection (Norm)*, using a normal distribution, and
- the *IPCC A1FI Projection (RC)*, using the raised-cosine distribution:

$$P(z') = \frac{1}{W}\left(1 + \cos\left(\frac{2\pi z'}{W}\right)\right) \quad \text{for} \quad -W/2 < z' < W/2 \quad \text{otherwise} \quad 0 \tag{5}$$

where *P(z')* is the full-width of the distribution. The raised-cosine distribution is generally the more realistic as it constrains the projections to finite upper and lower limits.

3.2. The post- AR4 Projections

Nicholls et al. (2011) summarised projections of sea-level rise published since the AR4 (their Table 1). They suggested ‘a pragmatic range of 0.5-2 m for twenty-first century sea-level rise, assuming a 4° C or more rise in temperature’. This temperature rise (which is for 2090-2099 relative to 1980-1999), is achieved by the AR4 temperature projections for emission scenarios A1B, A2 and A1FI. They also concluded that ‘the upper part of this range is considered unlikely to be realized’ (the 2 m upper limit of this range being derived from Pfeffer et_al. (2008)). It is also highly unlikely that sea level will fall under global warming. These considerations are here translated into a *‘21st century’* sea-level rise of 1.0 m ± 1.0 (lim) m, using a raised-cosine distribution function giving zero probability outside this range (Equ. 5).

A third set of sea-level rise allowances is based on this projection, which is here denoted the *1.0/1.0 m Projection* and applies to the ‘*21st century’*. This is roughly twice as large as the *IPCC A1FI Projection (Norm)* and the *IPCC A1FI Projection (RC)* for 1990-2100, both in mean and standard deviation.

3.3. Summary

Fig. 1 shows the distribution functions, *P(z'),* offset by the respective central values of the estimated rise (Δz) for the *IPCC A1FI Projection (Norm)* and the *IPCC A1FI Projection (RC)* for 1990-2100, and the *1.0/1.0 m Projection* for the *‘21st century’.*

4. APPLICATION OF THE METHOD

4.1. Allowances for Australia

The scale parameter (λ) was estimated from the *GESLA* (Global Extreme Sea-Level Analysis) sea-level database (see Menendez and Woodworth, 2010) which has been collected through a collaborative activity of the Antarctic Climate & Ecosystems Cooperative Research Centre, Australia, and the National Oceanography Centre Liverpool (NOCL), UK. The data covers a large portion of the world and is sampled at least hourly (except where there are data gaps). Only records longer than 30 years were used. Annual maxima were estimated using a declustering algorithm such that any extreme events closer than 3 days were counted as a single event, and any gaps in time were removed from the record. These annual maxima were then fitted to a Gumbel distribution using the *ismev* package (Coles, 2001). This yielded the scale parameter (λ) for each of the tide-gauge records. It is assumed that λ does not change in time.

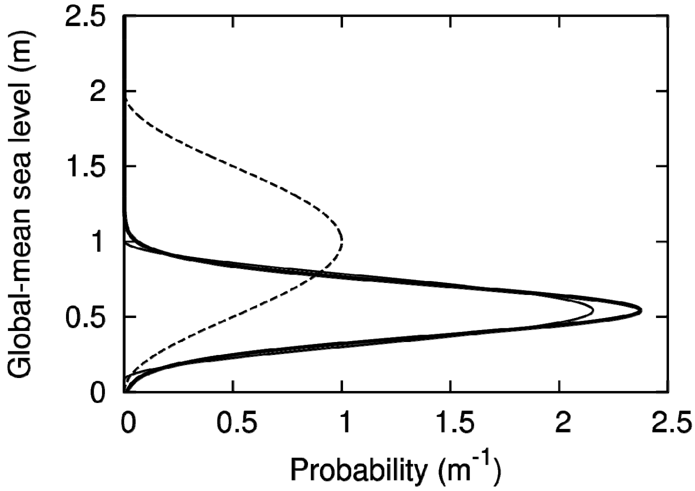

Figure 1: Distribution functions, P(z'), offset by the respective central values of the estimated rise (Δz). Thick and thin continuous lines indicate IPCC A1FI Projection (Norm) and IPCC A1FI Projection (RC), respectively, for 1990-2100. Thin dashed line indicates 1.0/1.0 m Projection for the '21st century'.

The results for Australia are here presented in three different ways. Firstly, the scale parameter indicates the way in which the frequency of extreme events changes for a given rise in mean sea level. From Eq. 3, a rise of mean sea level (δz) (which effectively increases the location parameter (μ) by δz) increases the expected number of exceedances (N) by a factor $\exp(\delta z/\lambda)$. This factor is shown (using the *left-hand key*) for a rise in mean sea level of 0.5 m in Fig. 2.

The other, and closely related, way of presenting the results is in terms of the sea-level rise allowances for different sea-level rise projections. Since both ways of presenting the results depend spatially only on the scale parameter (λ), they are here plotted in the same figure, but with different keys. Of the two 'IPCC' projections, the more conservative *IPCC A1FI Projection (Norm*) was chosen in deriving an allowance for 1990-2100 (however, as is shown in Section 4.2, the choice of distribution function (normal or raised-cosine) does not have a strong influence on the allowance during the 21st century). The allowances for the *IPCC A1FI Projection (Norm)* (for 1990-2100) and the *1.0/1.0 m Projection* (for the *'21st century'*) are shown by the *middle* and *right-hand* keys of Fig. 2, respectively.

For the long Australian GESLA stations, the scale parameter has a range of 0.06 – 0.19 m.

For the *IPCC A1FI Projection (Norm)* for 1990-2100, the sea-level rise allowance and its spatial variation are 0.68 ± 0.04 (sd) m. The average allowance represents a 26% increase over the mean sea-level rise of 0.54 m.

For the *1.0/1.0 m Projection* for the *'21st century'*, the sea-level rise allowance and its spatial variation are 1.44 ± 0.07 (sd) m. The average allowance represents a 44% increase over the mean sea-level rise of 1.0 m.

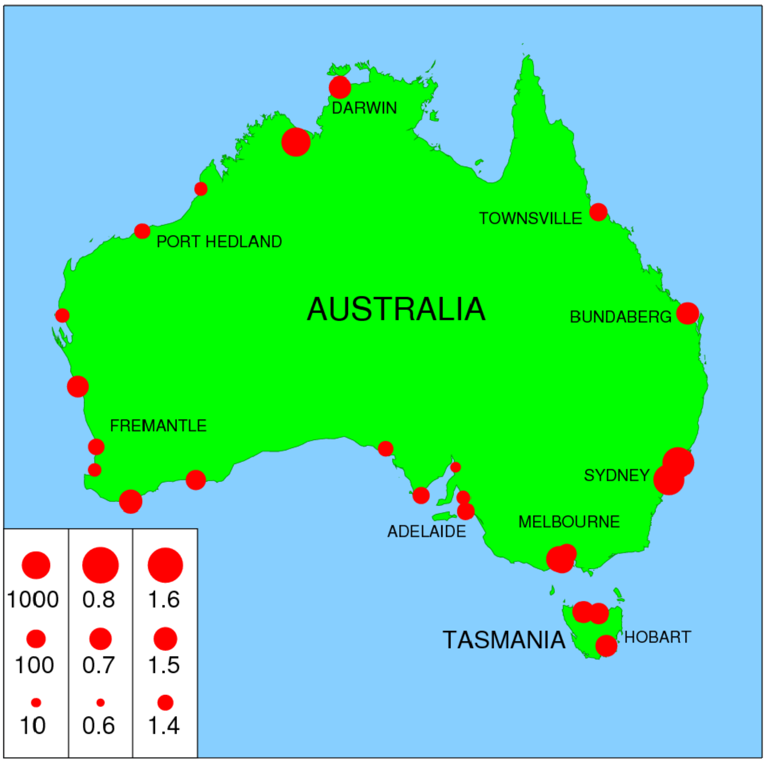

Figure 2: Results for Australia, indicated by dot diameter. (a) Factor by which frequency of flooding events will increase with a rise in sea level of 0.5 m; key is left-hand column of dots in the bottom left-hand corner. (b) Sea-level rise allowance (m) for 1990-2100 which preserves frequency of flooding events for IPCC A1FI Projection (Norm); key is central column of dots in the bottom left-hand corner. (c) Sea-level rise allowance (m) for '21st century' which preserves frequency of flooding events for the 1.0/1.0 m Projection; key is right-hand column of dots in the bottom left-hand corner.

4.2. The Temporal Evolution of the Allowance from 1990 to 2200

The temporal evolution of the allowance for the *IPCC A1FI Projection (Norm)* and the *IPCC A1FI Projection (RC)* is shown in Fig. 3*.* The projections were linearly extrapolated from 2100 to 2200, based on the gradient from 2090 to 2100. The allowance is calculated for Gumbel scale parameters of 0.05 and 0.20 m. This range includes over 95% of the long (> 30 year) global tide gauge records in the *GESLA* database; it also includes *all* of the long Australian records in the *GESLA* database.

Fig.3 shows that, for both scale parameters, the allowance follows closely the mean projection until about 2050 after which it moves towards the 95% (upper) limit. The allowance rises *faster* for locations with *smalle*r scale parameter, which is to be expected; a *small* scale parameter makes the number of extremes *more sensitive* to sea-level rise (differentiation of Equ. 3 yields $\partial N/\partial \mu = N/\lambda$).

For a given scale parameter, the allowance rises faster for the *IPCC A1FI Projection (Norm)* than for the *IPCC A1FI Projection (RC).* This is because the tails of the normal distribution are unbounded and therefore wider than the tails of the (bounded) raised-cosine distribution. This is particularly marked for the case of a scale parameter of 0.05 m, where the allowances at 2200 differ by a factor of about 1.5. The form of the distribution function selected for the sea-level rise projection is therefore crucial. A normal distribution is clearly unrealistic in the sense that there is an absolute limit to global sea-level rise: when all the water and ice stored on land has been transferred into the ocean. For a bounded distribution, it may be shown that the allowance asymptotes to the upper limit of the distribution.

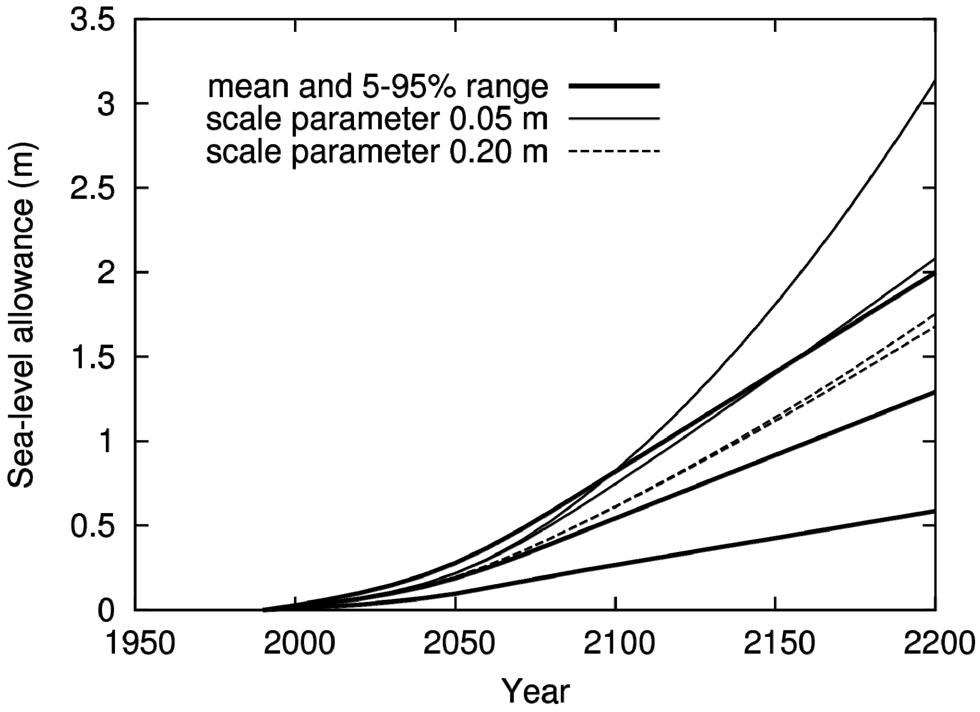

Figure 3: Sea-level rise allowance for 1990-2200. Thick continuous lines indicate the AR4-adjusted TAR projections (mean and 5-95% range) linearly extrapolated to 2200. Thin continuous lines indicate allowance for Gumbel scale parameter (λ) of 0.05 m (upper line is for IPCC A1FI Projection (Norm) and lower line is for IPCC A1FI Projection (RC)). Thin dashed lines indicate allowance for Gumbel scale parameter (λ) of 0.20 m (upper line is for IPCC A1FI Projection (Norm) and lower line is for IPCC A1FI Projection (RC)).

5. SUMMARY

A method has been described for estimating a vertical allowance for future sea-level rise. The allowance depends strongly on the distribution function describing the uncertainty in the projected rise (in particular, the width of the tails), but once this distribution has been chosen, the remaining derivation of the allowance is entirely *objective*. The allowance depends only on the Gumbel scale parameter (λ), the central value of the estimated rise (Δz), and the form and standard deviation (σ) of the distribution function describing the uncertainty in the rise.

The use of the allowance has been demonstrated in the Australian context, indicating 21st-century allowances of about 0.7 m for a projection based on the IPCC TAR and AR4, and about 1.4 m for a projection based on studies since the IPCC AR4. However, given the present uncertainties in the processes which determine sea-level rise, it must be emphasized that it is difficult to assign meaningful weights to these (quite different) projections.

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