# A Note on the Temperature Correction of the Aquatrak Acoustic Tide Gauge

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# Abstract

Temperature corrections for the Aquatrak acoustic tide gauge are derived. These account for variable temperature (and hence sound velocity) along the sounding tube, thermal expansion of the calibration tube and the thermal response of the transducer/controller combination. When averaged over long (e.g. seasonal) periods, the corrections are typically several millimeters in magnitude, with short (e.g. diurnal) period variations as large as several centimeters. The correction associated with thermal expansion of the calibration tube is of the same order as the correction for variable sound velocity, which is already applied at some Aquatrak installations. The corrections derived here relate to the situation where an initial adjustment has been made by the manufacturer (making the calibration tube a fixed known length under factory conditions), and a subsequent calibration is performed to define a single zero offset for the instrument.


# 1 Introduction

Acoustic tide gauges are now used for long-term high-accuracy observation of sea level (UNESCO 1994, 2002). In particular, the implementation of the Next Generation Water Level Measurement System (NGWLMS) in the Unites States has involved considerable development in this field over the past decade (e.g. Gill et al., 1993). Acoustic gauges are susceptible to small errors associated with temperature variations of the instrument. This note derives linear temperature corrections for one particular gauge (the Aquatrak), taking into account three processes:

1. the effect of variable air temperature (and hence sound velocity) along the sounding tube,
2. thermal expansion of the calibration tube (the part of the sounding tube above the calibration hole), and
3. the thermal response of the transducer/controller combination.

Process (1) and its correction has been discussed by Joseph et al. (1997) and Porter and Shih (1996). However, it is believed that processes (2) and (3) have not previously been considered and integrated into an analysis involving the calibration procedure. Aquatrak sensors may be driven by either a Sutron or a Bartex controller (the latter now being called an 'Aquatrak' controller). Correction terms have been calculated for both controllers.

The speed of sound in air depends on both temperature and relative humidity. In the present context, the effect of humidity is small compared with the effect of temperature (at 10°C, a change of 1°C has approximately the same effect on sound velocity as a 100% change in relative humidity; e.g. Pierce, 1981, Murray, 1967). This paper therefore considers only the effects of variable temperature.

# 2 Theory

A schematic of the sounding tube and acoustic transducer is shown in Figure 1. Since the components of the instrument are subject to thermal expansion, it is important to define the point at which the system is rigidly mounted. For most systems, and those considered here, the transducer is the mounting point, so that the transducer is located at a fixed height. The tide gauge measures the two-way travel time from the transducer to the calibration hole ($t_a$) and from the transducer to the sea surface ($t_b$). The Sutron and Bartex/Aquatrak controllers reference their outputs to the calibration hole and to the transducer, respectively. These travel times $t_a$ and $t_b$ are given by:

$$t_a = 2\frac{L_1}{c_1} \tag{1}$$

and

$$t_b = 2\left(\frac{L_1}{c_1} + \frac{L_2}{c_2}\right) \tag{2}$$

where $c_1$ and $c_2$ are representative sound velocities for the sections of sounding tube above and below the calibration hole, respectively.

It is assumed that the sound velocity, $c$, is primarily a function of temperature, so that

$$c_1 = c(T_1)\ ; \quad c_2 = c(T_2) \tag{3}$$

where $T_1$ and $T_2$ are representative temperatures for the sections of sounding tube above and below the calibration hole, respectively.

In the following analysis, only the Bartex/Aquatrak controller is considered, as indicated by the use of the subscript $B$. The analysis for a Sutron logger (indicated by the use of the subscript $S$) is very similar and is covered briefly in the Appendix.

The output of a Bartex/Aquatrak controller, $O_B$, is formed from:

$$O_B = \frac{t_b}{t_a} L_0(1 + \beta(T_e - T_*)) \tag{4}$$

where $L_0$ is a constant programmed by the factory to be an estimate of $L_1$, and $\beta$, $Te$ and $T_*$ describe the aggregated thermal response of the 'electronics' (the transducer/controller combination). $\beta$ is a temperature coefficient, $T_e$ is a representative temperature of the electronics and $T_*$ is the temperature at which the electronics has the correct gain (in the sense that, when $T_e = T_*$, the electronics outputs the exact value of $(t_b/t_a)L_0$). It should be noted that, in the absence of any thermally-induced errors in the electronics, $O_B - O_S = L_0$ (Equation (A1) in the Appendix, for a Sutron controller).

$L_1$ varies with temperature due to thermal expansion, and it is assumed that

$$L_1 = L_0 \tag{5}$$

when the temperature of this section of sounding tube (the calibration tube) is given by $T_0$.

When the calibration tube is at some other temperature, $T_1$

$$L_1 = L_0(1 + \alpha(T_1 - T_0)) \tag{6}$$

where $\alpha$ is a representative thermal expansion coefficient for the calibration tube. This is here assumed to be the thermal expansion coefficient of CPVC (chlorinated polyvinylchloride, of which the main part of the calibration tube is composed), although the component that is here defined as the 'calibration tube' also includes (non-CPVC) parts of the acoustic transducer. It should be noted that any error in the measurement of the length of the calibration tube may be included in our uncertainty of the value of $T_0$, as there is always some value of $T_0$ for which Equation (5) is correct.

Acoustic tide gauges are conventionally calibrated at a given temperature by defining the position of a 'calibration zero' such that:

$$O'_B = \Delta_B + L'_1 + L'_2 \tag{7}$$

where primed quantities relate to conditions during calibration (i.e. when the whole instrument is at some uniform temperature, $T_c = T_1' = T_2' = T_e'$). The acoustic path used during calibration is generally approximately the same as it is in the field ($L_2' \approx L_2$). It should be noted, with reference to Figure 1, that with the Sutron controller, the zero is approximately a distance $L_0$ below the transducer (i.e. near the calibration hole), while with the Bartex/Aquatrak controller, the zero is approximately at the transducer.

For a field observation, the distance $R_B$ of sea level below the calibration zero is given by

$$R_B = \Delta_B + L_1 + L_2 \tag{8}$$

Six small dimensionless quantities are defined:

$$\delta_s = 1 - \frac{c_2}{c_1} \tag{9}$$

$$\delta_{10} = \alpha(T_1 - T_0) \tag{10}$$

$$\delta_{c0} = \alpha(T_c - T_0) \tag{11}$$

$$\delta_{e*} = \beta(T_e - T_*) \tag{12}$$

$$\delta_{c*} = \beta(T_c - T_*) \tag{13}$$

$$\delta_L = 1 - \frac{L'_2}{L_2} \tag{14}$$

The quantity $\delta_s$ may be estimated by assuming a linear relationship between sound velocity, $c$, and absolute temperature, $T$ (Porter and Shih 1996):

$$\frac{\partial c}{\partial T} = \frac{c}{2T} \approx \frac{c}{550} \tag{15}$$

Equations (1), (2), (3), (4) and (13) may be used to derive $O'_B$ (the value of $O_B$ during calibration) in terms of $L_0$, $L'_1$, $L'_2$ and $\delta_{c*}$. This result and equations (6), (7) and (11) then yield:

$$\Delta_B = L'_2 \frac{\delta_{c*} - \delta_{c0}}{1 + \delta_{c0}} + L_0(\delta_{c*} - \delta_{c0}) \tag{16}$$

Equations (1), (2), (4), (9) and (12) may be used to derive an expression for $O_B$ in terms of $L_0$, $L_1$, $L_2$, $\delta_s$ and $\delta_{e*}$. This result and equations (6), (8), (10), (14) and (16) then yield

$$R_B = (O_B - L_0(1 + \delta_{e*}))\frac{(1 + \delta_{10})(1 - \delta_s)(1 + \delta_{c*} + \delta_L(\delta_{c0} - \delta_{c*}))}{(1 + \delta_{c0})(1 + \delta_{e*})} + L_0(1 + \delta_{10} + \delta_{c*} - \delta_{c0}) \tag{17}$$

It is now assumed that $\delta_s$, $\delta_{10}$, $\delta_{c0}$, $\delta_{e*}$ and $\delta_{c*}$ (but not $\delta_L$) are small and that their second-order terms may be neglected. Equation (17) then becomes

$$R_B = O_B - \underbrace{(O_B - L_0)\delta_s}_{\text{I}} + \underbrace{O_B(\delta_{10} - \delta_{c0})}_{\text{II}} - \underbrace{O_B(\delta_{e*} - \delta_{c*})}_{\text{III}} + \underbrace{(O_B - L_0)\delta_L(\delta_{c0} - \delta_{c*})}_{\text{IV}} \tag{18}$$

where correction terms on the right-hand side of Equation (18) are labeled I-IV.

The origins of the correction terms on the right-hand side of Equation (18) are:

**I:** variable temperature (and hence sound velocity) along the sounding tube,

**II:** thermal expansion of the calibration tube,

**III:** the thermal response of the transducer/controller combination, and

**IV:** an interaction between:

- the difference between the lengths of the acoustic paths for the calibration and for the field observations,
- the difference between the temperature of the calibration and the temperature at which $L_0$ is the exact distance between the transducer and the calibration hole,
- the difference between the gain of the transducer/controller combination during calibration, and the 'true' gain.

Term I is the correction term described by Porter and Shih (1996). It depends on the sound velocities $c_1$ and $c_2$, which are estimated from the temperatures $T_1$ and $T_2$, which in turn are generally recorded at an Aquatrak installation.

Term II depends on the temperature difference $T_1 - T_c$. $T_1$ is generally recorded at an Aquatrak installation, while $T_c$ should be recorded during calibration.

Term III depends on the representative temperature of the transducer/controller combination, $T_e$. The transducer temperature is generally not recorded, but will probably be close to the temperature of the calibration tube, $T_1$. The temperature of the controller is generally recorded at an Aquatrak installation. Term III also depends on $T_c$, which should be recorded during calibration, and on $T_*$ and $\beta$, neither of which is presently well known.

Term IV depends on the temperatures $T_c$, $T_0$ and $T_*$, which are fixed for a given installation, and on $\delta_L$ which varies with the acoustic range. This term is poorly known, due to uncertainties in $T_0$, $T_*$ and $\beta$. Since no information is available on the value of $\delta_{c*}$, the subsidiary Term IV′ is defined as the value of Term IV with $\delta_{c*} = 0$:

$$\text{Term IV}' = (O_B - L_0)\delta_L \delta_{c0} \tag{19}$$

Since Term I depends on the temperature difference $T_1 - T_2$, it is zero in experiments carried out in an environment of uniform temperature. Aquatrak Corporation have carried out such experiments on a system with ($O_B = 2.5$ m, $L_0 = 1.2$ m) and concluded that, over a temperature range of 19 - 29°C in which $T_1 - T_2$ was maintained approximately constant and less than 0.3°C, the output of the Bartex/Aquatrak controller, $O_B$, changed by less than 0.0003 m (Luis Ponce of Aquatrak Corporation, 2002, personal communication). During this experiment, Term II should vary as $O_B \alpha T_1$ (from equations (10) and (18)), and hence change by 0.0016 m (see next section for the value of $\alpha$), significantly larger than the observed variation in controller output. The thermal characteristics are therefore better than would be expected if the terms in equation (18) behaved independently, significant cancellation among the Terms I, II and III being the most probable cause. If Term II were to be substantially canceled by Term I, then $T_1 - T_2$ would have to change by about 0.6°C (using equation (15)), which it clearly did not. It therefore seems probable that much of Term II is canceled by Term III.

In the following section, approximate values of Terms I, II and IV$'$ are derived for a specific installation.

## 3 An Example

An Aquatrak gauge and Bartex/Aquatrak controller has been installed at Port Arthur, Tasmania, since 1999. The gauge is mounted in a small hut on a jetty, with the transducer about 3.2 meters above mean sea level. The stilling well is mounted on a northward-facing side of the jetty, and hence is warmed by the sun at certain times of the day, especially during the summer. This warming leads to a temperature difference along the sounding tube which is generally around 1°C but may at times reach 9°C.

From a two-year tidal record taken from Port Arthur, Tasmania, during 1999 to 2001, using an Aquatrak gauge and Bartex/Aquatrak controller, the following values are typical. For the 'small' quantities, $T_1 - T_2$, $\delta_s$, $\delta_{10}$ and $\delta_{c0}$, estimates of the root mean square values are given. Limits indicated by '±' are estimates of the standard deviations. Any signs preceding the following values have been ignored.

$$O_B \approx 3.2 \text{ (m)}$$

$$L_0 \approx 1.2 \text{ (m)}$$

$$L_2 \approx 2.0 \text{ (m)}$$

$$L_2' \approx 3.6 \text{ (m)}$$

$$T_1 = 13 \pm 5 \text{ (}^\circ\text{C)}$$

$$
\begin{aligned}
T_2 &= 13 \pm 5 \ (^\circ\mathrm{C}) \\
T_1 - T_2 &\approx 1.2 \ (^\circ\mathrm{C}) \\
T_0 &= 22 \pm 3 \ (^\circ\mathrm{C}) \\
T_c &= 22 \pm 3 \ (^\circ\mathrm{C}) \\
\alpha &\approx 0.000063 \text{ (for CPVC) } (^\circ\mathrm{C}^{-1}) \\
\delta_s &\approx (T_1 - T_2)/550 \approx 0.002 \\
\delta_{10} &\approx 0.0006 \\
\delta_{c0} &\approx 0.0003 \\
\delta_{10} - \delta_{c0} &= \alpha(T_1 - T_c) \approx .0006 \\
\delta_L &\approx 0.8
\end{aligned}
$$

The correction terms are therefore (again with signs ignored):

$$
\begin{aligned}
\text{Term I}: &\quad (O_B - L_0)\delta_s \approx \quad 0.004 \text{ (m)} \\
\text{Term II}: &\quad O_B(\delta_{10} - \delta_{c0}) \approx \quad 0.002 \text{ (m)} \\
\text{Term IV}': &\quad (O_B - L_0)\delta_L\delta_{c0} \approx \quad 0.0005 \text{ (m)}
\end{aligned}
$$

Figures 2 and 3 show Terms I and II for periods in mid-summer (1999/2000) and mid-winter (2000), derived from the Port Arthur data.

# 4 Discussion

The correction terms, I-III, on the right-hand side of Equation (18) are approximately proportional to the length of the acoustic path (Term IV depends on

$(O_B - L_0)\delta_L \approx L_2 - L_2'$, which should be small). For the example given here, the acoustic path is relatively small. For many tide gauge installations, the transducer is significantly higher above the water and the required correction terms would be correspondingly larger.

In this example, the correction is dominated by Term I, which depends on variable temperature along the sounding tube. This term is generally dominated by direct solar heating of parts of the sounding tube and therefore varies over the diurnal heating/cooling cycle, often changing sign during the day. This correction may hence be reduced by averaging (e.g. in an estimation of mean sea level), but would also contribute to solar tidal constituents such as $S_1$ and $S_2$. The temperature difference in the above example, $(T_1 - T_2)$, is also only an RMS value (1.2°C). This temperature difference may often be as large as 5°C, which, using the example from Port Arthur, would require a correction term of 0.02 meters.

Term II, which depends on thermal expansion of the calibration tube, is in most cases negative, because the outdoor air temperature is generally cooler than the indoor temperature at which the calibration is carried out (in the above example, by around 9°C). This correction is therefore largely *systematic* throughout the record, and largest in winter.

Unknown quantities in the above analysis are the temperature, $T_0$, at which the length of the calibration tube equals $L_0$, the temperature, $T_*$, at which the gain of the transducer/controller combination is correct, and the thermal sensitivity, $\beta$, of that gain. As regards $T_0$, the manufacturer adjusts the physical length of the calibration tube to match $L_0$, but the temperature at which this exercise is carried out is unknown (although it is probably close to $T_c$, given that both this adjustment and the calibration procedure are generally carried out indoors). $T_0$ contributes only to Term IV. The remaining unknowns, $T_*$ and $\beta$, contribute to Terms III and IV. As noted earlier, it is probable that there is significant cancellation of Term II by Term III. Since it is not at present possible to estimate Term III, it can only be included in the final error estimate. It is an open question whether Term II should be included explicitly as a correction, or included in the final error estimate.

It is probable that Term IV is relatively small, since the effects of thermal expansion of the calibration tube and the thermal sensitivity of the transducer/controller combination are believed to be comparable, and Term IV′ is the smallest of the terms estimated. Term IV is therefore probably best ignored as an adjustment, but included in the final error estimate.

This paper has described temperature corrections which may be applied to an Aquatrak acoustic tide gauge. It is also necessary to make some estimate of the error in the corrected range. The above discussion considers the most common case in which only two temperature measurements are made along the sounding tube, one of the calibration tube and the other of the remainder of the sounding tube. These measurements clearly cannot be truly representative of the average temperatures in each section of the sounding tube, and they also may not be representative of the temperature of the air *inside* the sounding tube (the temperature sensors are generally attached to the outside of the sounding tube). Joseph et al. (1997) concluded that, at one site, correction for the sound velocity variation along the sounding tube only reduced the error by a factor of about three. The error due to this source is therefore typically (Term I)/3. On the other hand, temperature variations of the calibration tube itself are likely to be smaller than variations of temperature of the air in the sounding tube, so any correction for thermal expansion of the calibration tube (Term II) is probably quite robust, and we can neglect any error from this source. Terms III and IV are poorly known and should hence be included in the error estimate.

The corrections associated with variable sound velocity (Term I) and with thermal expansion of the calibration tube (Term II) are of similar size. When averaged over long (e.g. seasonal) periods, the corrections are typically several millimeters in magnitude, with short (e.g. diurnal) period variations as large as several centimeters. One of these corrections (Term I) is already applied at some Aquatrak installations. The magnitude of these corrections may be viewed in the context of the present rate of sea level change. The Intergovernmental Panel on Climate Change have estimated that the global average rate of sea level rise over the 20th century was 1 to 2 mm/year (Church et al., 2001) and it is believed that records of 50 to 80 years duration are required in order to provide meaningful estimates of such changes (Douglas, 2001). In this context, a viable sea level record would involve a total rise of around 100 mm. It is therefore prudent to attempt to constrain all errors associated with sea level measurements to the millimeter, rather than to the centimeter level.

# 5 Acknowledgments

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# 6 Appendix

## The Sutron Controller

This Appendix describes the changes that should be made to the previous derivations when the Aquatrak tide gauge is driven by a Sutron controller, rather than a Bartex/Aquatrak controller.

For a Sutron controller, equation (4) becomes

$$O_S = \frac{t_b - t_a}{t_a} L_0 (1 + \beta(T_e - T_*)) \tag{A1}$$

equation (7) becomes

$$O'_S = \Delta_S + L'_1 + L'_2 - L_0 \tag{A2}$$

equation (8) becomes

$$R_S = \Delta_S + L_1 + L_2 - L_0 \tag{A3}$$

equation (16) becomes

$$\Delta_S = L'_2 \frac{\delta_{c*} - \delta_{c0}}{1 + \delta_{c0}} - L_0 \delta_{c0} \tag{A4}$$

equation (17) becomes

$$R_S = O_S \frac{(1 + \delta_{10})(1 - \delta_s)(1 + \delta_{c*} + \delta_L(\delta_{c0} - \delta_{c*}))}{(1 + \delta_{c0})(1 + \delta_{e*})} + L_0(\delta_{10} - \delta_{c0}) \tag{A5}$$

equation (18) becomes

$$R_S = O_S - O_S\delta_s + (O_S + L_0)(\delta_{10} - \delta_{c0}) - O_S(\delta_{e*} - \delta_{c*}) + O_S\delta_L(\delta_{c0} - \delta_{c*})$$

$$\qquad\qquad \text{I} \qquad\qquad \text{II} \qquad\qquad\qquad \text{III} \qquad\qquad\qquad \text{IV} \qquad\qquad \text{(A6)}$$

and equation (19) becomes

$$\text{Term IV}' = O_S\delta_L\delta_{c0} \tag{A7}$$

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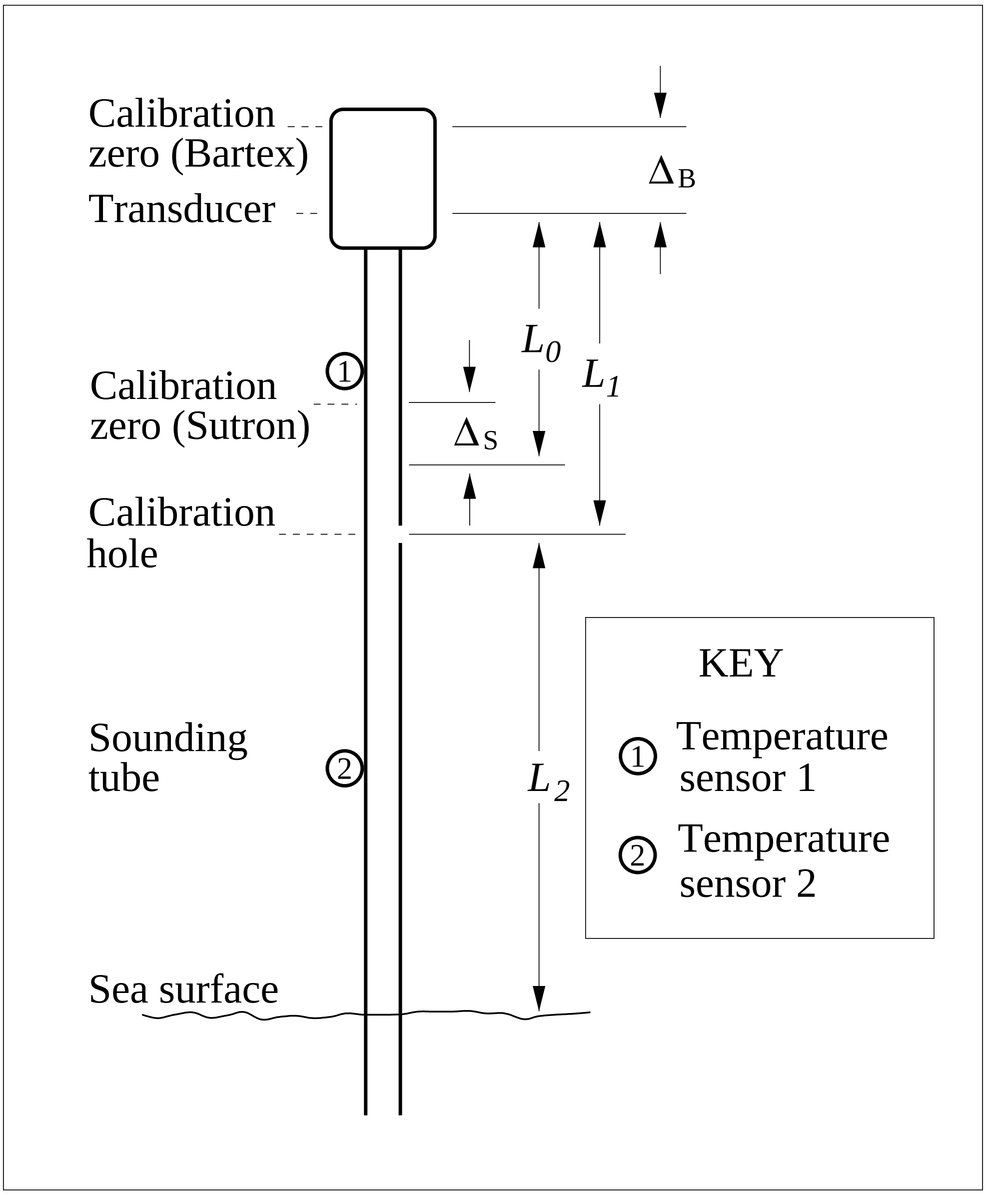


Figure 1: Schematic of an acoustic tide gauge. $L_1$ and $L_2$ are the actual lengths of the sections of sounding tube above and below the calibration hole, respectively. $L_0$ is a constant programmed by the factory to be an estimate of $L_1$. $\Delta_S$ and $\Delta_B$ are calibration constants for the Sutron and Bartex controllers, respectively. The instrument outputs an estimate of the distance between the relevant calibration zero and the water surface.

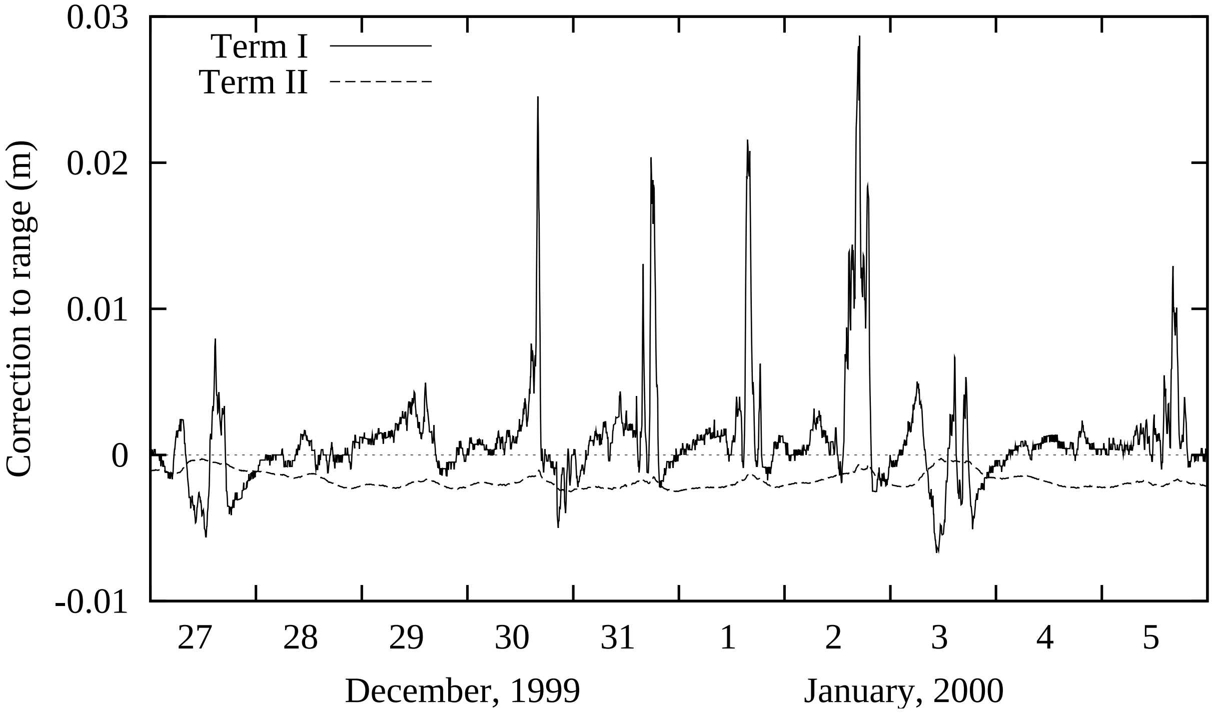


Figure 2: Temperature corrections to range, mid-summer (1999/2000). Dates are in local time.

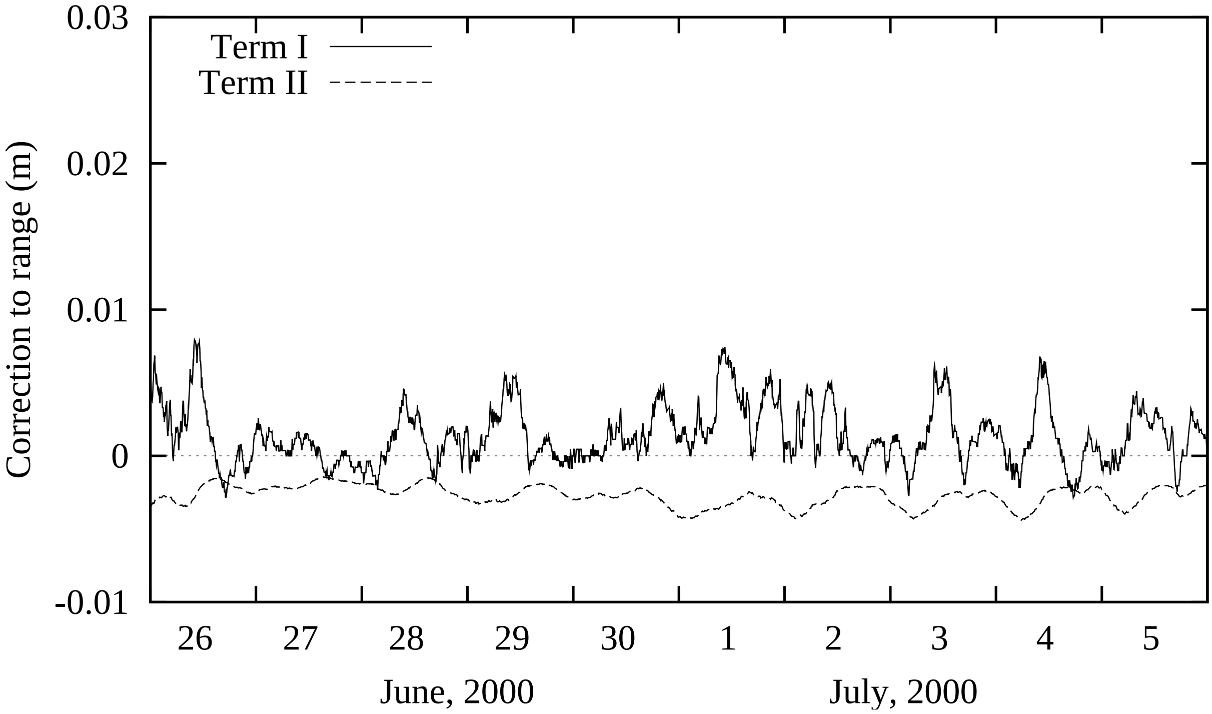


Figure 3: Temperature corrections to range, mid-winter (2000). Dates are in local time.

## 8 Figure Captions

1. Schematic of an acoustic tide gauge. $L_1$ and $L_2$ are the actual lengths of the sections of sounding tube above and below the calibration hole, respectively. $L_0$ is a constant programmed by the factory to be an estimate of $L_1$. $\Delta_S$ and $\Delta_B$ are calibration constants for the Sutron and Bartex controllers, respectively. The instrument outputs an estimate of the distance between the relevant calibration zero and the water surface.

2. Temperature corrections to range, mid-summer (1999/2000). Dates are in local time.

3. Temperature corrections to range, mid-winter (2000). Dates are in local time.